III. "On the Thermal Effect of drawing out a Film of Liquid." By Professor WILLIAM THOMSON, F.R.S., &c., being extract of two Letters to J. P. JOULE, LL.D., F.R.S., dated February 2 and 3, 1858. Received April 30, 1858.

A very novel application of Carnot's cycle has just occurred to me in consequence of looking this morning into Waterston's paper on Capillary Attraction, in the January Number of the Philosophical Magazine. Let T be the contractile force of the surface (by which in Dr. Thomas Young's theory the resultant effect of cohesion on a liquid mass of varying form is represented), so that, if Π be the atmospheric pressure, the pressure of air within a bubble of the liquid of radius r, shall be $\frac{4T}{r}+\Pi$. Then if a bubble be blown from the end of a tube (as in blowing soap-bubbles), the work spent, per unit of augmentation of the area of one side of the film, will be equal to 2T.

Now since liquids stand to different heights in capillary tubes at different temperatures, and generally to less heights at the higher temperatures, T must vary, and in general decrease, as the temperature rises, for one and the same liquid. If T and T′ denote the values of the capillary tension at temperatures t and t' of our absolute scale, we shall have $2(T-T')$ of mechanical work gained, in allowing a bubble on the end of a tube to collapse so as to lose a unit of area at the temperature t and blowing it up again to its original dimensions after having raised its temperature to t'. If $t'-t$ be infinitely small, and be denoted by $\mathfrak{T}$, the gain of work may be expressed by

$$-\frac{2dT}{dt}\times\mathfrak{T};$$

and by using Carnot's principle as modified for the Dynamical Theory, in the usual manner, we find that there must be an absorption of heat at the high temperature, and an evolution of heat at the low temperature; amounting to quantities differing from one another by

$$\frac{1}{J}\times\frac{-2dT}{dt}\times\mathfrak{T},$$

and each infinitely nearly equal to the mechanical equivalent of this

difference, divided by Carnot's function, which is $\frac{\mathrm{J}}{t}$, if the temperature is measured on our absolute scale. Hence if a film such as a soap-bubble be enlarged, its area being augmented in the ratio of 1 to m, it experiences a cooling effect, to an amount calculable by finding the lowering of temperature produced by removing a quantity of heat equal to

$$m\frac{t}{\mathrm{J}}\times\frac{-d\mathrm{T}}{dt},$$

from an equal mass of liquid unchanged in form.

For water T=2·96 gr. per lineal inch.

Work per square inch spent in drawing out a film =5·92, say 6 grains, $\frac{d\mathrm{T}}{dt}=\frac{1}{550}\mathrm{T}$, or thereabouts.

Suppose $\frac{t'}{\mathrm{J}}=\frac{300}{1390\times12}$, then the quantity of heat to be removed, to produce the cooling effect, per square inch of surface of augmentation of film will be $\frac{1}{5100}$. Suppose, then, 1 grain of water to be drawn out to a film of 16 square inches, the cooling effect will be $\frac{16}{5100}$ of a degree Centigrade, or about $\frac{1^{\circ}}{320}$. The work spent in drawing it out is $16\times6=96$ grains and is equivalent to a heating effect of $\frac{96}{12\times1390}=\frac{1}{174}$. Hence the total energy (reckoned in heat) of the matter is increased $\frac{1}{174}+\frac{1}{320}$ of a degree Centigrade, when it is drawn out to 16 square inches.

IV. "On the Logocyclic curve, and the geometrical origin of Logarithms." By the Rev. J. Booth, LL.D., F.R.S. Received April 15, 1858.

In a paper read before the Mathematical Section of the British Association during its meeting at Cheltenham in 1856, and which was printed among the reports for that year, I developed at some length the geometrical origin of logarithms, and showed that a trigonometry exists as well for the parabola as for the circle, and that every formula in the latter may be translated into another which shall indicate some property of parabolic arcs analogous to that from which it has been derived. I showed, moreover, that the